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ANALYSIS TOOLS FOR THICK CONCRETE SLABS[1]

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In some applications, such as transfer plates in multistory buildings, or mat foundations, the structural thickness of concrete slabs used is much larger than the common residential or commercial floors. When using post-tensioning, it is not uncommon to select a span to depth (L/h) ratio of 45 for a residential building, while for transfer plate of the same building this ratio may have to be reduced to as low as 3 to support and distribute the weight of floors above it. Figure 1 shows an example of a transfer plate supporting the weight of over 40 floors, where span to depth ratio is approximately 4.

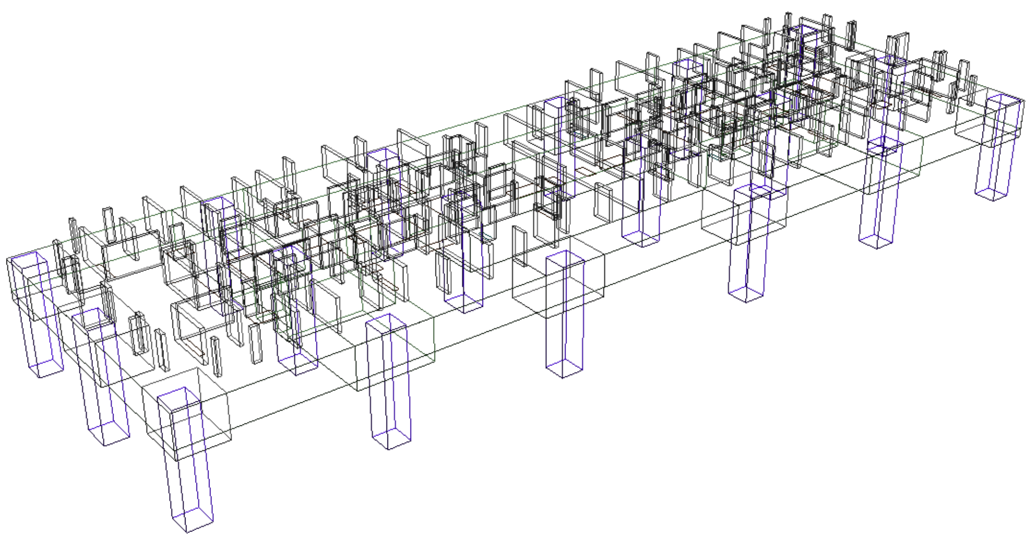

FIGURE 1 VIEW OF A TRANSFER PLATE SUPPORTING SHOWING THE LOADS FROM ABOVE AND THE SUPPORTS BELOW

This Technical Note reviews the behavior of thick concrete slabs used as transfer plates, and evaluates the applicability of the current analysis tools for their design. It concludes that while the response of transfer plates is somewhat complex and depends on a number of factors normally not included in the common analysis schemes, the tools available today provide a safe and serviceable design. In other words, the code specified satisfactory in-service response and safety of transfer plates can be established using the proper commercially available tools.

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STRUCTURAL DESIGN OBJECTIVES

The primary objectives in the structural design of a transfer plate are:

- Satisfactory in-service response (SLS)
  This translates to (i) control of deflections and (ii) crack width. Due to their small span to depth ratio (L/h) vibration and bounciness do not govern thick plate design.
- Adequate Safety against over load (ULS)
  This involves two steps, namely: (i) the accurate or conservative determination of design values (demand moment, shear, etc) at each design and (ii) that at each section adequate reinforcement is provided to achieve a capacity in excess of design demand.

While the exact deflection of a transfer plate and the value of stresses within the depth of a transfer plate are of interest, they do not impact the design process, provided the primary design objectives as stated in the preceding are met.

BACKGROUND

B and D Regions and Design Values

Figure 2 illustrates the correlation in span (L) to depth (h) variation of plate strips with the development of D regions in concrete slabs. The B regions represent where the underlying assumption for analysis of thin plates applies, namely while deforming, plane sections remain plane and normal to the centroidal axis of the member. This assumption leads to a linear distribution of strain across the depth of plate sections. The regions marked with D extend between one to two times the depth of the section from the supports. The distribution of strain within the D regions is more complex. It is non-linear. Shear strains, location and details of the support within the depth of the member play significant role in distribution of stress in D regions.

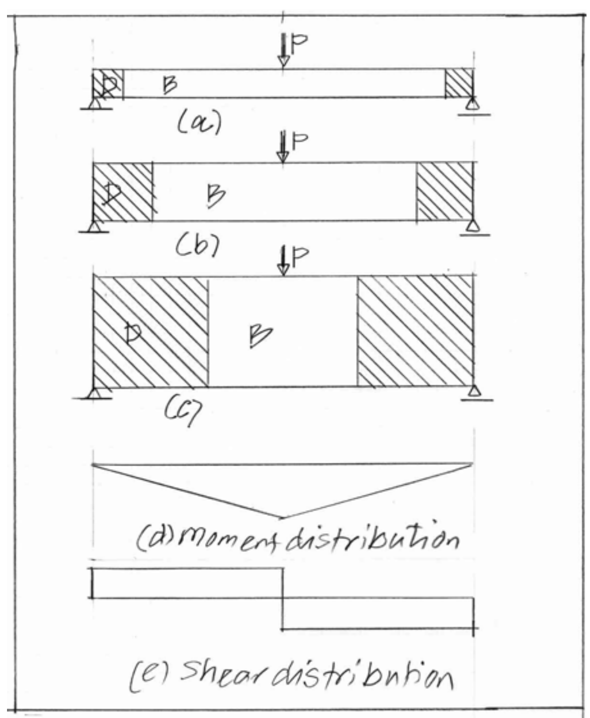

FIGURE 2 MOMENT AND SHEAR FOR MEMBERS WITH DIFFERENT L/h RATIOS
(L=span; h=member depth)

From standpoint of structural design, it is noteworthy that for the statically determinate structure shown in Fig. 2, the design values of moment and shear are the same for all L/h ratios. Since the Ultimate Limit State (ULS) design of a section is based on the post-cracking response, and for shear design the codes clearly recognize the proximity of sections next to supports (D regions), it follows that using the local design values and following the recommendations of building codes on the design and detailing of sections will meet the safety requirements, irrespective of L/h ratio. The conclusion is highlighted in Fig. 3-c, where M, V and N are the same for widely different L/h ratios shown in parts (a) and (b) of the figure, and the schematic of their design procedure shown in parts (d) and (e) of the same figure.

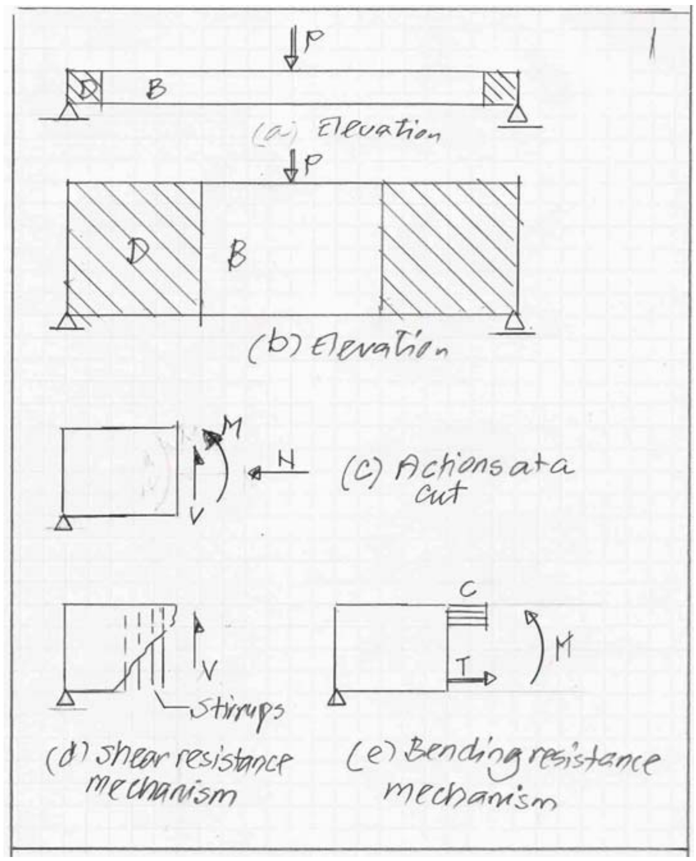

FIGURE 3 DESIGN VALUES AND SCHEMATICS OF DESIGN RESISTANCE

Thin Plate Theory

Thin plate theory is used in many commercially available software to analyze and design floor systems. The following describes the salient features of thin plates that impact the design of thick plates.

Figure 4 shows the elevation of a member under uniform load. The end supports are located below the member. The supports can be either pins (fixed in position, but rotationally free), or roller (fixed against vertical displacement, but free to translate horizontally). When using “thin plate theory,” the structure is modeled with a zero plate thickness (line in the figure) located at the centroid of the member. The supports are assumed to have been located at the centroidal axis as shown in part (b). The stresses at each section along the centroidal axis are determined from the rotation of normals to the centroidal axis. The displacement of the centroidal axis is solely due to the curvature (ψi) caused by bending of the sections. Two items of the plate response are not accounted for - namely (i) the presence of shear and its contribution to the displacement of the member, and (ii) the impact of the support locations with respect to the centroidal axis.

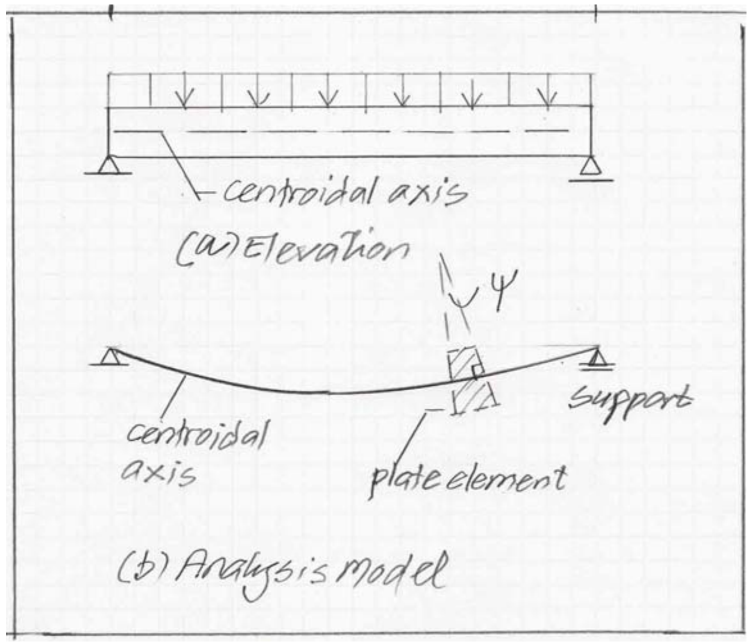

FIGURE 4 DISPLACEMENT USING SIMPLE BEAM THEORY

For low values of L/h the contributions of shear and arch action in resisting the applied loads become more significant. Likewise, ratio of deflection due to shear compared to that of bending becomes larger. The following is intended to quantify the shear deflection, in order to evaluate its significance in practical design scenarios.

The distribution of vertical shear stresses across the depth of a plate is shown in Fig. 54-(b-i) in form of a parabola with the maximum value at plate's center equal to 1.5 times the average stress (τ) shown in (b-ii) of the same figure.

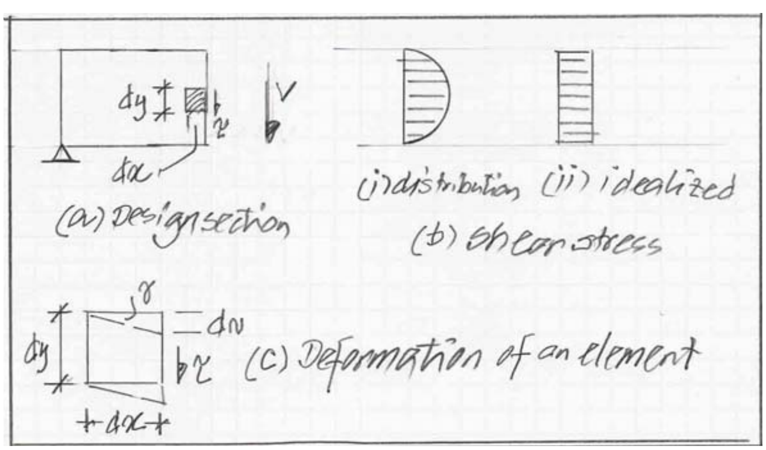

FIGURE 4 SHEAR DEFLECTION MECHANISM

The deformation of an infinitesimal element of the plate is $dv = \gamma * dx$, where $\gamma = \tau/G$ (G is the shear modulus). It can be shown (see appendix A) that for a uniformly distributed load on a slab strip the ratio of deflection due to bending (δ_b) to that of shear (δ_v) is given by:

$$\delta_b / \delta_v = 0.52 (L/h)^2$$

The impact of shear deflection for uniform loading on a plate strip is shown in Fig. 6. Observe that for a design ratio of L/h = 4 and uniform load, the deflection from bending alone is 8.32 times the associated deflection due to shear. In other words, for a span to depth ratio of 4, the simple plate theory can underestimate the calculated deflection by approximately 12%. For low span to depth ratios deflection does not govern the design, and if needed, it can be magnified by the calculated ratio before comparing the deflection with the code specified requirements.

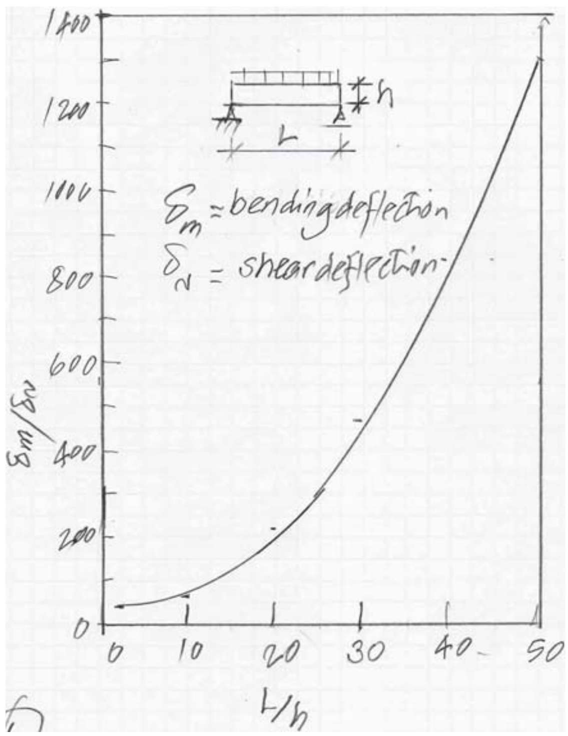

FIGURE 6 BENDING TO SHEAR DEFLECTION RATIOS FOR A PLATE STRIP

Extended Thin Plate Theory

As illustrated in Fig. 7, for plates with finite thickness the arch action between the point of application of the load and the supports acts as a viable load path in addition to the traditional bending and shear transfer discussed for thin plate theory. The contribution of the arch action depends in the first instance on the span to depth ratio (L/h) of the plate, and next to the location and fixity of the supports.

Observe the phenomenon illustrated in Fig. 8. For thin plates, such as L/h=35, the positional fixity of the supports results in development of a net tensile force in the plate – part (a) of the figure. The horizontal tensile force at the support leads to an increase in the design moment of the plate (part c of the figure). The amount of increase depends on the fixity of the support. On the other hand, for low span to depth ratios, such as L/h = 10, the positional fixity of the supports leads to the development of net compression in the plate and a reduction in the design moment. (see parts b and c of the figure).

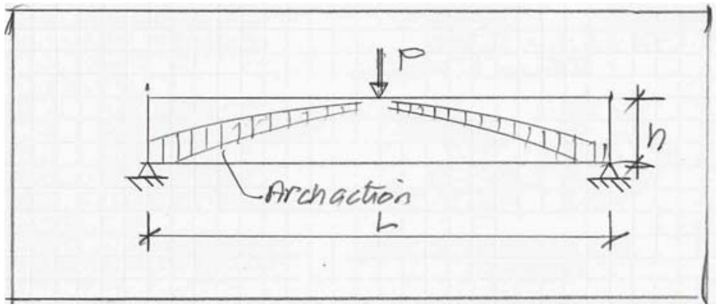

FIGURE 7 ARCH ACTION IN PLATES WITH FINITE THICKNESS

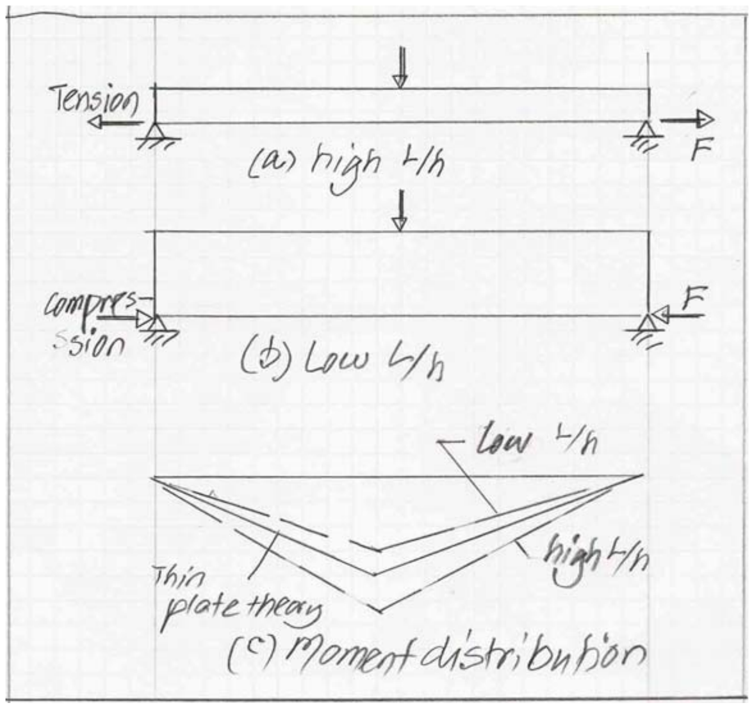

FIGURE 8 HORIZONTAL REACTIONS FOR DIFFERENT SPAN TO DEPTH RATIOS AND THEIR IMPACT ON DESIGN VALUES

The extended thin plate theory, as formulated in ADAPT-Floor Pro software recognizes the plate thickness, the position of each support with respect to the centroidal axis of the plate, the degree of fixity provided at each connection to the support and duly accounts for the development of horizontal forces at the supports and their impact on the design values.

The quantitative impact of positional fixity of the supports on the design moment of a plate under uniform loading is shown in Fig. 9. Note that for roller supports there is no difference between the moments calculated by thin plate theory and the extended thin plate theory. However, once the fixity of supports is accounted for, the design moments differ to an extent that makes them design significant.

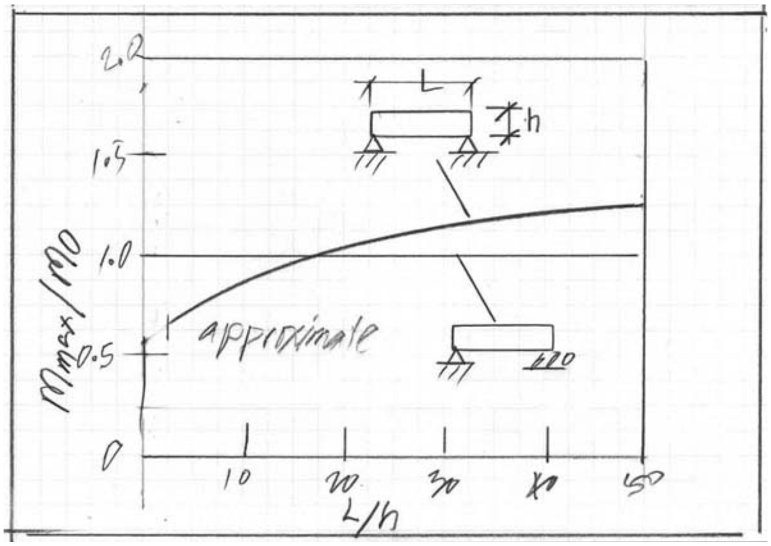

FIGURE 9 IMPACT OF POSITIONAL FIXITY OF SUPPORTS ON DESIGN MOMENTS OF A PLATE STRIP.

Thick Plate Analysis

Thick plate analysis includes the combination of arch action, fixity of the supports and contribution of shear deformation to deflection of plates. The primary difference between the thick plate analysis and the extended thin plate is the impact of shear deformation on plate deflection. The contribution of shear deformation was quantified in the preceding (Fig. 6). For lower values of span to depth ratios the thin plate theory underestimates the maximum deflection as illustrated in Fig. 10 for two extreme boundary conditions.

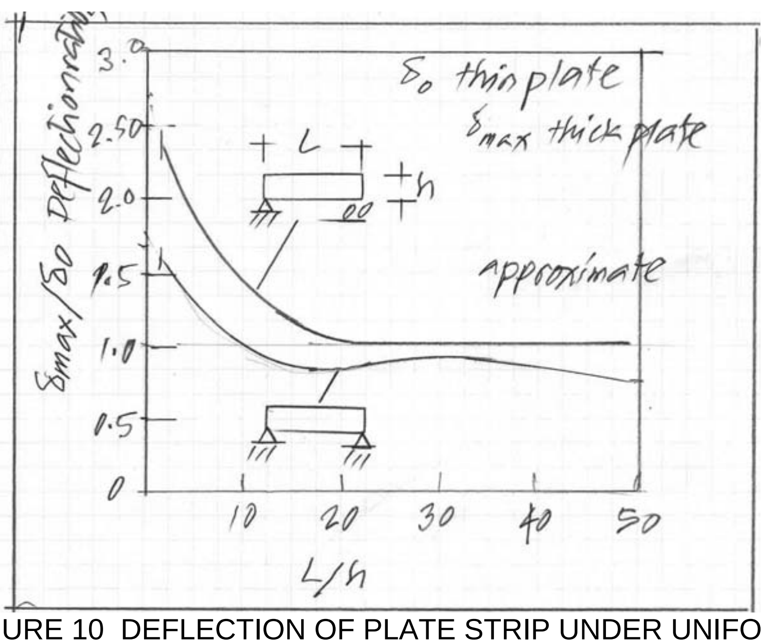

FIGURE 10 DEFLECTION OF PLATE STRIP UNDER UNIFORM LOAD USING THICK PLATE ANALYSIS.

CONCLUDING REMARKS

From the analysis results presented and discussed above, the following conclusions are derived.

- The design values determined using extended thin plate analysis are valid for the safety (ULS) design and evaluation of thick plates.
- The deflections determined by extended thin plate analysis do not include the contribution of shear deformation. While shear deformation is determined not to be critical in practical design, strictly speaking, the deflections calculated by the extended thin plate theory have to be magnified by the coefficients given in graph of Fig. 6.
- The location and the restraint provided by the supports (fixity of the supports) have a design significant influence in the design of transfer plates. The support conditions should be recognized in detail and accounted for in the analysis.

ADAPT-Floor Pro computer program is based on the extended thin plate theory. The results produced by this program account for the impact of a plate's depth, the position and restraint of the supports. The accuracy in deflections obtained from ADAPT-Floor Pro can be improved, if these are multiplied by the coefficients obtained from Fig. 6.

APPENDIX A

Details of Data Used in the Analysis Results Presented in a separate Technical Note

APPENDIX B

Discretization of Extended Thin Plate, and Thick Plates used for Results Presented in This Technical Note.

All plate strips analyzed were 10.00 long. The thickness was varied from 200mm to 2500 mm to cover the range of span to depth ratio from 50 to 2.50. In each instance the plate was discretized through its depth into a number of finite elements to create a relatively square finite element cell. Figure C-1 is the illustration of the plate analyzed for span to depth ratio of 4.

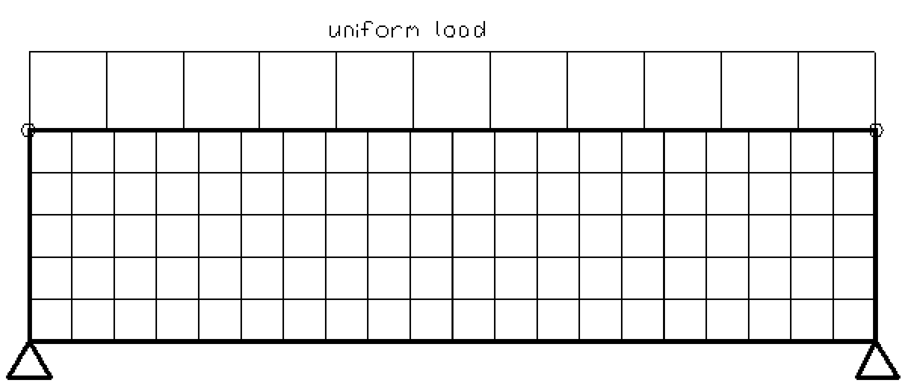

FIGURE B-1 DISCRETIZATION OF THICK PLATE FOR L/h=4